TERTIARY ENTRANCE EXAMINATION, 2000

QUESTION/ANSWER BOOKLET

APPLICABLE MATHEMATICS

Please place your student identification label in this box

STUDENT NUMBER - In figures

In words ....

To be completed by candidates

What kind of ***graphics*** calculator did you bring to this examination?

Make and model:

1. ....

2. ....

None ☐ (tick if applicable)

TIME ALLOWED FOR THIS PAPER

Reading time before commencing work: Ten minutes
Working time for paper: Three hours

MATERIAL REQUIRED/RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR

This Question/Answer Booklet

TO BE PROVIDED BY THE CANDIDATE

Standard Items: Pens, pencils, eraser or correction fluid, ruler

Special Items: Curriculum Council SEA Mathematical Formulae and Statistical Tables Book, drawing instruments, templates, notes on two sheets of A4 paper and calculators satisfying the conditions set by the Curriculum Council.

Note: Personal copies of the Tables Book should not contain any handwritten or typewritten notes, symbols, signs, formulae or any other marks (including underlining and highlighting), except the name and address of the candidate, and may be inspected during the examination.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room.

It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor BEFORE reading any further.

Ref: 404

STRUCTURE OF THIS PAPER

Questions	Marks Available
1	6
2	5
3	4
4	6
5	6
6	5
7	12
8	11
9	6
10	8
11	6
12	9
13	10
14	3
15	6
16	6
17	13
18	16
19	8
20	6
21	14
22	6
23	8

Total marks = 180

INSTRUCTIONS TO CANDIDATES

ALL questions should be attempted. You may answer questions in any order you wish.

Write your answers in the spaces provided.

An extra page is supplied on the back of this booklet. If the extra page is used, label the questions carefully. Indicate on the original question that your working continues at the end of this workbook.

Show all working clearly, in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

On the front cover you are asked to state the kind of graphics calculator that you brought into the examination. This information is required for research to monitor that fair outcomes are achieved. Please ensure that you complete this box. Please note that the same marking procedure will apply to all scripts, whatever calculators have been used.

At the end of the examination, check that your Student Identification Label and your Student Number (in figures and words) have been placed in the spaces provided on the front cover of this Question/Answer Booklet.

1. (6 marks)

For the matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & -3 \\ -1 & 2 & 4 \end{bmatrix},$$

calculate the following, giving detailed reasons if the calculation is not possible.

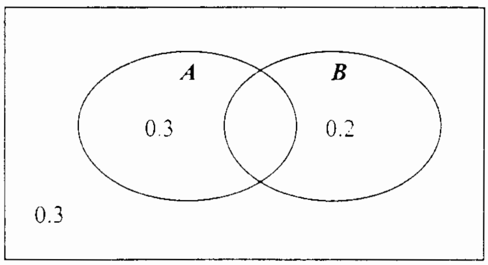
(i) $A + 2B$ [2]

(ii) $C + D$ [2]

(iii) A^{-1} [2]

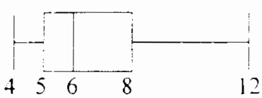
2. (5 marks)

By examining the probabilities given in the Venn diagram below, determine whether or not events A and B are independent.



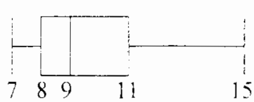
3. (4 marks)

The set, X, of data illustrated on the box and whisker plot below has a mean of 7 and a variance of 5.



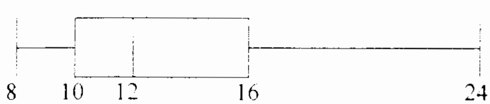
Two different linear transformations of the original data are illustrated below. Write down the mean and variance for each new set of data.

(i)



[2]

(ii)



[2]

4. (6 marks)

Susie bought a 3 hour videotape to record a film to be shown on TV. The brand of videotape she chose has a length of tape that is normally distributed with a mean of 185 minutes and a standard deviation of 2 minutes.

(a) What percentage of this brand of videotape have lengths less than three hours? [2]

(b) Including all the credits, the film that Susie wants to record is three hours and seven minutes long. What is the probability that Susie can get all of the film and credits on her tape? [2]

(c) Suppose the credits at the end of the film are one minute long and Susie has managed to record all of the film up to the start of the final credits. What is the probability that she will now be able to record all of the credits as well? [2]

5. (6 marks)

Four consecutive customers visiting an outback caravan shop bought bread (loaves), milk (litres), newspaper(s) and petrol (litres). The quantity of each item bought by each customer and their total cost (dollars) are represented by the matrix system of equations below, where b, m, n and p are the cost of one loaf of bread, one litre of milk, one newspaper and one litre of petrol respectively.

$$\begin{bmatrix} 2 & 1 & 1 & 15 \\ 3 & 3 & 2 & 10 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 20 \end{bmatrix} \begin{bmatrix} b \\ m \\ n \\ p \end{bmatrix} = \begin{bmatrix} 21.45 \\ 21.50 \\ 3.90 \\ 26.70 \end{bmatrix}$$

(a) Indicate clearly how a method using inverse matrices can be used to find the costs of the individual items. [2]

(b) Write down the appropriate inverse matrix. [2]

(c) Find the cost of each individual item. [2]

6. (5 marks)

Given the following system of equations

$$\begin{aligned} x - 2y + z &= 1 \\ 2x + y - z &= 3 \\ 3x - y &= \boldsymbol{a} \end{aligned},$$

describe the solution set when $\boldsymbol{a} = 4$ and the solution set when $\boldsymbol{a} = 5$. Justify your answer.

7. (12 marks)

A Swedish study on sets of twins found that the probability of identical twins was $\frac{1}{3}$ and the probability of non-identical twins was $\frac{2}{3}$. For a set of identical twins the probability of a set of either sex is the same. For sets of non-identical twins the birth of a boy is as equally likely as the birth of a girl.

(a) Illustrate the probabilities for all possible combinations of twins on a tree diagram. [4]

For each of the following questions assume that a set of twins from this study is chosen at random.

(b) Determine the probability that the twins consist of

(i) two boys, [2]
(ii) two children of the same sex. [2]

(c) If the twins are both girls, what is the probability that they are identical? [2]

(d) If at least one of the twins is a girl, what is the probability that they are identical? [2]

8. (11 marks)

A small airline operates out of Sydney. Below is part of a spreadsheet which shows the monthly passenger kilometres from January 1997 to December 1999. Appropriate centred moving averages (CMAs) are shown.

For **prediction** the following model is appropriate:

passenger kilometres = trend + seasonal component

The **trend** (found using the CMAs) is given by the least squares regression line:

$$N(t) = 7.360 + 0.0223\, t$$

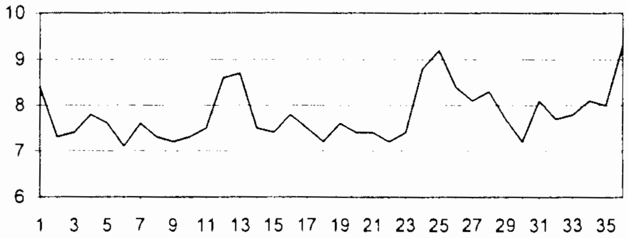
This has been used to calculate the predicted passenger kilometres on the spreadsheet.

		t	Passenger km (millions)	CMA	Predicted Passenger km $N(t)$ (millions)
January	1997	1	8.4		7.38
February	1997	2	7.3		7.41
March	1997	3	7.4		7.43
April	1997	4	7.8		7.45
May	1997	5	7.6		7.47
June	1997	6	7.1		7.49
July	1997	7	7.6	7.60	7.52
August	1997	8	7.3	7.63	**A**
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
December	1999	36	**B**		8.16

Residuals are calculated using the trend and some are given in the following table:

Month	Residuals 1997	1998	1999
⋮	⋮	⋮	⋮
September	–0.36	–0.43	–0.30
October	–0.28	–0.65	–0.02
November	–0.11	–0.47	–0.14
December	0.97	0.90	1.14

The following is a **graph** of the recorded passenger kilometres versus time:



(a) Use the graph on the previous page to determine the length of the cycle. [1]

(b) Comment upon the pattern indicated by the trend. [1]

(c) Use the trend to find the average annual increase in passenger kilometres. [2]

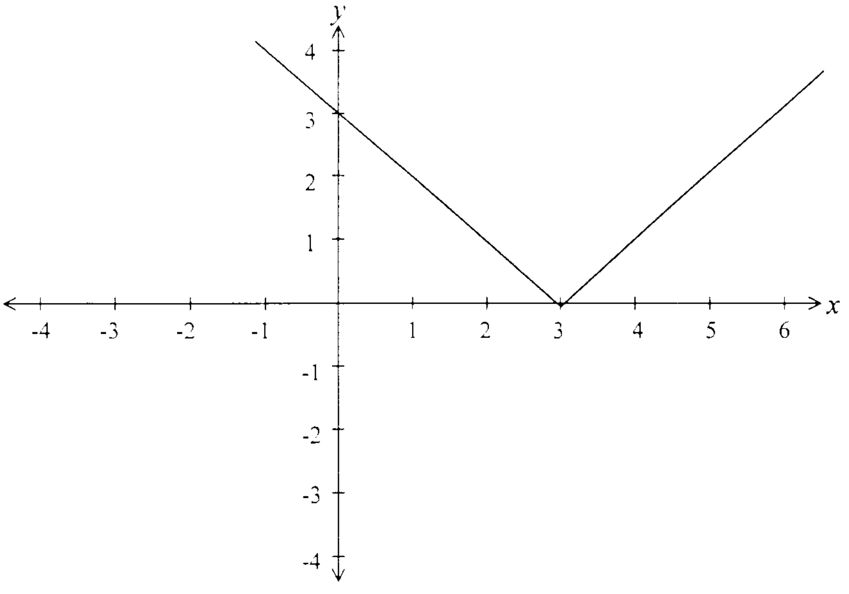
(d) Find the missing values labelled **A** and **B** in the displayed data. [3]

(e) Calculate the seasonal component for December. [2]

(f) How many passenger kilometres are expected for December 2000? [2]

9. (6 marks)

Given below is the graph of $f(x-2) = |x-3|$, where $y = f(x-2)$.



On the set of axes above, indicating clearly which graph is which, sketch the graphs of

(i) $y = f(x-2) - 3$, [2]

(ii) $y = -f(x-2)$, [2]

(iii) $y = f(x)$. [2]

10. (8 marks)

A household kept the following record of local calls made from their home telephone for a period of six months.

Month	Total Number of Calls	Total Duration of Calls (mins)
1	35	153
2	86	280
3	38	140
4	41	201
5	50	250
6	33	151

(a) For the six-month period, calculate

(i) the average duration of a local call, [1]
(ii) the standard deviation for the number of calls per month. [2]

(b) Suppose that local calls are charged at a rate of 5 cents per minute and the telephone connection charge for a month is $5. Find the average telephone bill per month for the household, assuming that only local calls are made. [3]

(c) The number of local calls in a particular month can be thought of as being unusually large if the number for the month is more than two standard deviations larger than the average number per month for the six-month period. Justifying your answer, list any months having an unusually large number of local calls, given that the average number of calls per month for the six-month period is 47.17. [2]

11. (6 marks)

In a company with 141 staff, 120 of the staff have a desktop computer on their office desk, 40 have a laptop computer and 33 have a desktop computer at home provided by the company. Six of the staff have all three types of computing setup, whilst 87 have only one type. Fourteen of the staff have both a laptop and a home computer but no desktop computer on their office desk. All staff who have a laptop computer have a desktop computer either at home or at work.

Construct a Venn diagram to display clearly how many staff are provided with what type(s) of computing setup.

12. (9 marks)

The data below gives the **total** kilometres travelled by passenger vehicles and the **average** kilometres travelled by passenger vehicles in a particular country between the years 1979 and 1998.

Year t	Total km travelled K (in millions)	Average km travelled (in thousands)
1979	84.9	15.8
1982	96.1	15.3
1985	106.6	15.4
1988	116.6	15.1
1991	114.3	14.5
1995	123.7	14.4
1998	134.3	14.2

(a) Calculate the correlation coefficient between the time, t, and the total kilometres travelled, K. [2]

(b) Give the least squares linear regression line relating total kilometres travelled to time. [2]

(c) Predict the total kilometres travelled in the year 2005. Comment upon the reliability of your prediction. [2]

(d) Without performing any calculations, explain whether or not the correlation coefficient between time and the average kilometres travelled will have the same sign as the correlation coefficient between time and total kilometres travelled. Suggest what this might indicate about the number of vehicles. [3]

13. (10 marks)

(a) Transform the set of coordinates P_1 (1, 2), Q_1 (0, 2), R_1 (0, −2) to the new set of coordinates P_2, Q_2, R_2 using matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. Write down the new coordinates. [2]

(b) Transform the set of coordinates P_2, Q_2, R_2 to the new set of coordinates P_3, Q_3, R_3 using matrix $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$. Write down the new coordinates. [2]

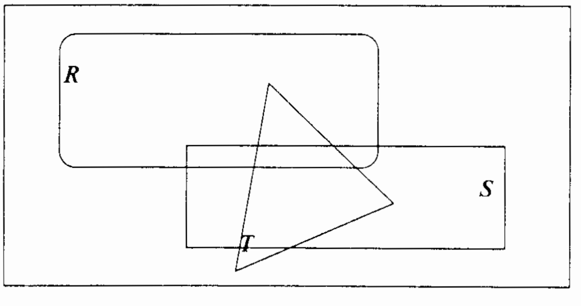
(c) Find **the matrix** that transforms the set of coordinates P_3, Q_3, R_3 back to the original set of coordinates P_1, Q_1, R_1. [3]

(d) If each of the sets of coordinates form the vertices of a triangle, find the ratio of the area of triangle $P_3 Q_3 R_3$ to the area of triangle $P_1 Q_1 R_1$. [3]

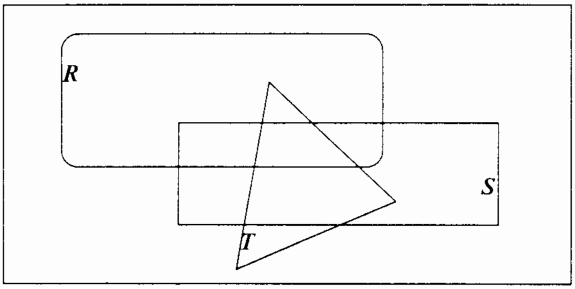
14. (3 marks)

Shade the indicated areas on the following Venn diagrams.

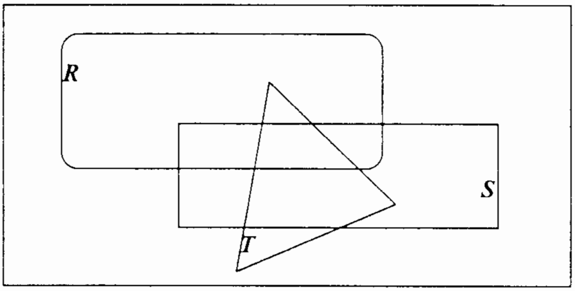
(i) $R \cup (S \cap T)$ [1]



(ii) $\overline{R} \cap S \cap T$ [1]



(iii) $(R \cup S) \cap (S \cup T)$ [1]



15. (6 marks)

(a) Calculate AC and BC for the following matrices:

$$A = \begin{bmatrix} 3 & 3 \\ -2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}.$$ [3]

(b) Given any three square matrices D, E and X that are all the same size, under what conditions will the following statement be true?

"If $DX = EX$ then $D = E$."

Justify your answer. [3]

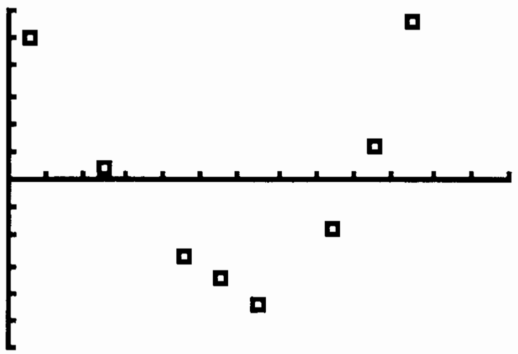
16. (6 marks)

The data below represent the size, p (in thousands), of the population aged over 65 for a small country between the years 1896 and 1995. Values are for the end of the year.

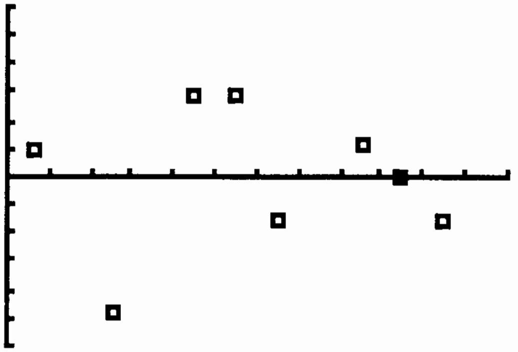
Year	Over 65 population, p
1896	151
1915	231
1935	415
1945	535
1955	660
1975	1128
1985	1438
1995	1827

A statistician fitted both a linear model and an exponential model to the data. In order to decide which was the most appropriate model, the statistician compared the residual plots for the linear model and the log-linear model equivalent to the exponential model. These residual plots are given below.

Residual Plot for Linear Model



Residual Plot for Log-Linear Model equivalent to Exponential Model



(a) Is the linear model or the exponential model more appropriate for the data? Justify your answer. [2]

(b) Find the equation for the model you chose as more appropriate in (a). [2]

(c) If the total population for the country at the end of 1996 was 20.8 million, find the proportion of over 65's, assuming the observed trend continued. [2]

17. (13 marks)

The owner of an antique shop in a busy part of town is wondering whether he needs to employ more staff. Currently he works alone. Many people wander in and just look at the antiques without buying. However, the owner finds that he has on average 60 paying customers (ie customers that do buy) in a 40 hour week. He can chat to several people at a time but feels that he has to give more time to the paying customers than to those just looking. The owner can usually handle about two paying customers in an hour. From his understanding of probability, the owner knows that the number of paying customers arriving at his shop closely models a Poisson distribution.

(a) What is the average number of paying customers per hour? [1]

(b) What is the probability that two paying customers arrive in any given one hour period? [1]

(c) What is the probability that he will have more paying customers than he can handle in any given one hour period? [2]

(d) It is financially viable to employ an extra person if there is more than a 30% chance of more than two paying customers arriving in any one hour period. Should he employ extra help? [1]

Under the model's assumptions, the time between the arrival of two consecutive paying customers follows an exponential distribution.

(e) Find the average time, in minutes, between the arrival of two paying customers. [2]

(f) Suppose the owner takes a 20 minute break to go to the bank. What is the probability that the next paying customer will not arrive until after the owner has returned? [3]

(g) Having waited for an hour since the last paying customer arrived, what is the probability that there will be at least one paying customer in the next 30 minutes? [3]

18. (16 marks)

In a certain year, Mrs Chan taught Calculus in one school and Applicable Mathematics in another school. At the end of the year she summarised the percentage marks (rounded to nearest whole number) obtained by the students in the two classes. This is given in the frequency table below.

% mark range	Calculus	Applicable
0 – 19	1	2
20 – 39	3	2
40 – 59	3	5
60 – 79	9	11
80 – 99	5	7

(a) Find the mean and standard deviation for the Calculus marks. [3]

(b) If a student from one of these classes is picked at random, what is the probability that the student

(i) received a mark greater than or equal to 60, [2]

(ii) was in the Applicable Mathematics class, [1]

(iii) was in the Calculus class and received a mark ≥ 80, [1]

(iv) was in the Calculus class, if the student had received a mark between 60 and 79 inclusive? [2]

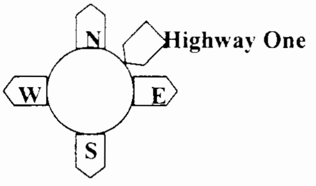
(c) Suppose that 4 students are selected at random from between the two classes. Find the probability that at least 1 of them has marks ≥ 80. [3]

(d) A survey carried out in the school where Mrs Chan taught Calculus indicated that a quarter of the students walk to school. Whether or not a particular student walks to school can be considered independent of whether or not another student walks to school.

If six students are chosen at random from this school, calculate the probability that at least three of them walk to school. [4]

19. (8 marks)

Highway One comes into the city at a roundabout. Four other roads (North, East, South and West) lead off the roundabout.



During a two hour period on a weekday morning a study was made of how many of the 6000 vehicles entering the roundabout from Highway One left via each of the other four roads. (No vehicles were observed that went all the way round and left by Highway One.)

South Road was the most used with twice as many vehicles leaving the roundabout via it as the combined number of vehicles leaving via North Road and East Road. The least busy was East Road which was used by only one third the number of vehicles that used West Road. Five hundred more vehicles left the roundabout via North Road than via East Road.

Find a system of four equations in four unknowns to model the above study. Write the system in the matrix form (not as an augmented matrix). DO NOT SOLVE THE SYSTEM BUT DO REMEMBER TO DEFINE YOUR VARIABLES.

20. (6 marks)

Chris and Kim were both playing computer games which have a playing time limit of 3.5 minutes. Chris's score, after playing for t minutes, was given by the value of the function $c(t)$ whilst Kim's score was given by the value of the function $k(t)$. If Chris and Kim started playing at the same instant, give the range of times (to the nearest one hundredth of a minute) when Chris had a lower score than Kim given that the functions are defined as follows.

$$c(t) = \frac{-(t+4)}{(t-4)} \qquad\qquad k(t) = \frac{t(t+3)(5-t)}{4}$$

Indicate clearly the method you use to answer this question.

21. (14 marks)

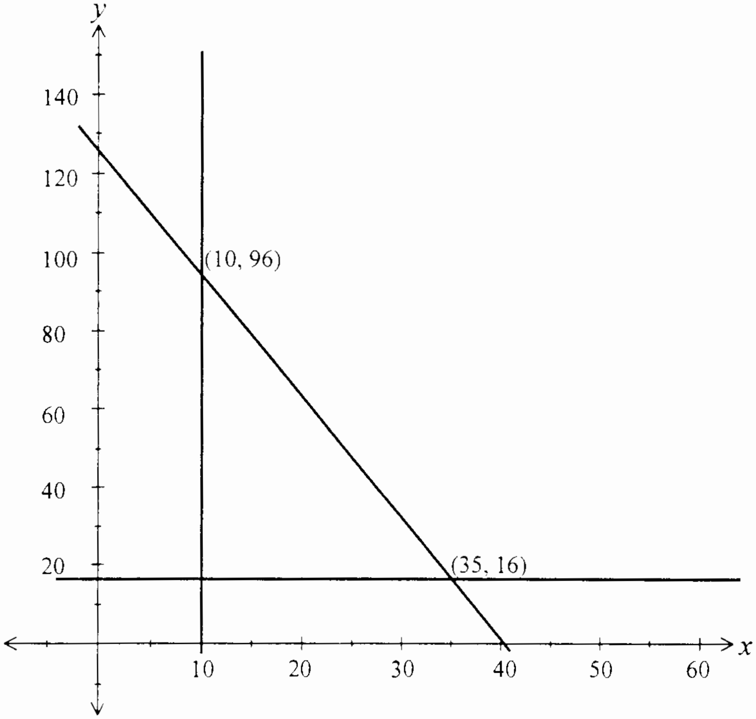
A factory has a contract to manufacture exclusive merino wool knitwear. A trial suggests that jackets will take 3.2 hours each and vests will take one hour each to machine knit. The jackets will each take 48 minutes to assemble, label, check and package, whilst the vests will each take 30 minutes.

The factory has 128 hours of available machine hours for knitting a week and 40 hours a week for assembling, labelling, checking and packaging. At least 10 jackets and 16 vests must be produced each week to fulfil a regular order.

Each jacket can be sold at a profit of $100 and each vest can be sold at a profit of $40.

Let x be the number of jackets and y be the number of vests produced in a week.

Some of the above constraints are drawn on the following graph.



(a) Draw the remaining constraint(s) and shade the feasible region. [4]

(b) Assuming all jackets and vests that are manufactured are sold, state how many of each should be manufactured in a week for a maximum profit. Justify your answer and state the maximum profit. [5]

A new design of jacket will cost less to produce but can be sold for the same amount as the previous jacket, thus profits will increase. The factory still wants to maintain its regular orders but switch to production of the new design.

(c) What is the increase in the profit per jacket that will give the most flexibility in the number of vests and jackets manufactured, whilst still maintaining orders and maximising profits (assuming all are sold)? State all possible combinations of numbers of vests and jackets that could be manufactured (and sold) for the new maximum profit. [5]

22. (6 marks)

The time taken for an individual to respond to a particular stimulus is anywhere between one and twenty seven seconds. The probability that an individual takes t seconds to respond can be obtained from the following probability density function, f.

$$f(t) = \begin{cases} \dfrac{(t-1)^2}{2}e^{-(t-1)} & 1 \le t \le 27 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate the probability that an individual responds in

(i) at most 15 seconds, [2]

(ii) at least 7 seconds, [2]

(iii) between 2.6 and 9 seconds. [2]

23. (8 marks)

Consider a large number, n, of independent events each having an outcome that can be classified as a success or failure and each having the same probability, p, of success.

HYPOTHESIS: *When determining relevant probabilities, if p is small, the Poisson distribution provides a better approximation to the binomial distribution than the normal distribution does.*

Suppose it is appropriate to use a binomial distribution for determining probabilities associated with the number, X, of individuals suffering from a particular disease. Suppose also that in a population of 900 the probability of an individual suffering from the disease is 0.01.

The probability that between 8 and 12 (inclusive) individuals in the population have the disease is determined to be 0.55415 when the binomial distribution is used.

(a) Use the normal approximation to the binomial distribution to determine the probability that between 8 and 12 (inclusive) individuals in the population have the disease. Specify clearly which normal distribution you are using. [3]

(b) Use a Poisson distribution with the same mean as your normal distribution in (a) to determine the probability that between 8 and 12 (inclusive) individuals in the population have the disease. Remember that the Poisson distribution is a discrete distribution like the binomial distribution and not a continuous distribution like the normal distribution. [3]

(c) Indicate whether or not your results indicate support for the hypothesis given at the beginning of this question. Justify your answer. [2]

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Dr Gopal Nair